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ABSTRACT

This study investigated the effects of a series of mathematics lessons which were designed to encourage a spatial mode of thinking. Using fifth-grade students from four intact classrooms, a pretest and posttest battery composed of spatial tests, reasoning tests, and a problem-solving test, were administered to the experimental and two control classes as a measure of the effects of the instruction. Results showed that mathematical problem-solving performance increased significantly, particularly for students of low reasoning ability. (Author/MM)

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THE EFFECTS OF SPATIAL INSTRUCTION ON MATHEMATICAL PROBLEM-SOLVING PERFORMANCE

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ABSTRACT

This investigation sought to analyze the effects of a series of lessons which were designed to encourage a spatial mode of thinking. Brain hemispheric studies have shown that such lessons are needed in order to exercise the spatial hemisphere of the brain. Using fifth-grade students from four intact classrooms, a pretest and posttest battery, composed of spatial tests, reasoning tests and a problem-solving test, were administered to the two experimental and two control classes as a measure of the effects of the instruction. Mathematical problem-solving performance increased significantly, particularly for students of low reasoning ability.

THE EFFECTS OF SPATIAL INSTRUCTION ON MATHEMATICAL PROBLEM-SOLVING PERFORMANCE

The improvement of the problem-solving performance of children has been a recognized objective of the elementary school mathematics curriculum. Researchers have devoted a considerable amount of time to the variables which affect the problem-solving process. Characteristics of the problem (e.g., vocabulary, sentence structure, the number of pieces of relevant information, and types of efficient solution processes) (Kane 1970; Krutetskii, 1976) have been studied; characteristics of the problem solver (e.g., intelligence, sex, level of mathematical maturity, reading level and cognitive styles) have been considered (Chase, 1960; Eastman, 1975). Recent research has focused on what appears to be the core of the problem: what are the most effective ways of teaching problem solving in the elementary classroom and are these different ways affected by the characteristics of the problem and the characteristics of the problem solver? Although there are no formulas which can be followed to a guaranteed successful venture in the problem-solving domain, are there any instructional strategies which appear to be promising?

Current research studies concerning brain hemispheric activity (Wheatley, 1977) have indicated to educators that an insufficient amount of time is being devoted to the development of the right brain hemisphere which is responsible for spatial thinking activities. Most public school instruction dwells on algorithmic, sequential activities. The child is taught to follow some methodical procedure in order to

arrive at the solution to problem-solving tasks. Rather than trying to obtain a holistic view of the problem, the child immediately attempts to apply some algorithm. He has no visual sensation of the problem as a whole.

The question arises as to whether an instructional program could be devised which would help elementary school children to use the right hemisphere and to solve mathematical problems using these right hemisphere processes. If, indeed, problem solving is a creative activity and if, indeed, the right hemisphere is the location of creative thought, then will practice in using the right hemisphere (through spatial activities) improve the problem-solving performance of elementary school children?

Purpose of the study

This investigation attempted to analyze the effects of a series of lessons which were designed to encourage a spatial mode(right hemisphere) of thinking. The following general questions represent the focus of the investigation:

(1) What effect, if any, will spatial instruction have on the mathematical problem-solving performance of elementary school students?

(2) What effect, if any, will spatial instruction have on the solution processes used in mathematical problem solving?

(3) Is there any significant aptitude-treatment interaction, that is, will the instruction affect students of high spatial ability differently than students of low spatial ability and will it affect students of high reasoning ability differently than students of low

reasoning ability?

(4) Is there any significant sex-treatment interaction, that is, will the instruction affect males differently than females?

Selection and description of the population sample

As is common in most investigations carried out in public schools, random assignment of students to control or experimental groups was impossible. Intact classrooms were used. This procedure has its advantages and disadvantages. Disadvantages include the loss of a large number of degrees of freedom in the statistical analyses. Advantages include the avoidance of certain threats to external validity. In particular, by randomly assigning students to groups and not using intact classrooms, a new and artifical environment is created which may bias results.

Two mathematics teachers at Newburgh Elementary School in Newburgh, Indiana were identified as willing to aid in this investigation. The two teachers taught a total of four fifth-grade mathematics classes; one teacher taught three classes and the other teacher taught one class. These four classrooms constituted the sample population for the investigation. Two of the four classes (taught by the first teacher) were randomly selected as experimental classes; the remaining two were designated as control classes.

A total of 132 students were involved in the study from the four classes. Most of the students were from middle-class families. Approximately 50% of the students were bused into school.

The choice of the fifth-grade level was made for several reasons. One major consideration was that it is more interesting to study the problem-solving processes of students who have been exposed to all four basic operations and have solved some word problems. Such students can solve a wider variety of problems and will be using a wider variety of techniques in their solution processes. Another reason for selecting fifth-grade students concerned the general learning strategies used by students. As has been previously mentioned, public school instruction is generally rule-oriented and sequential. Older students (e.g. high school students) would have had that much more of this rule-oriented instruction than would younger students. The effects of a brief series of lessons involving the spatial mode of thinking, such as this investigation proposed, would probably be minimal on students who had received so much rule-oriented instruction prior to the investigation. It was therefore decided that fifth-grade students could profit the most from the short-term spatial instruction that was involved in this study.

Procedure

The study was carried out over a 15-week period during the second semester of the 1977-78 academic year. The three phases which made up the study are illustrated in Figure 1. During the first week of the study, all pretest evaluation instruments were administered to both the experimental and the control classes. During the following 13 weeks, the two experimental classes received the spatial instruction presented by the investigator one day per week during the regular

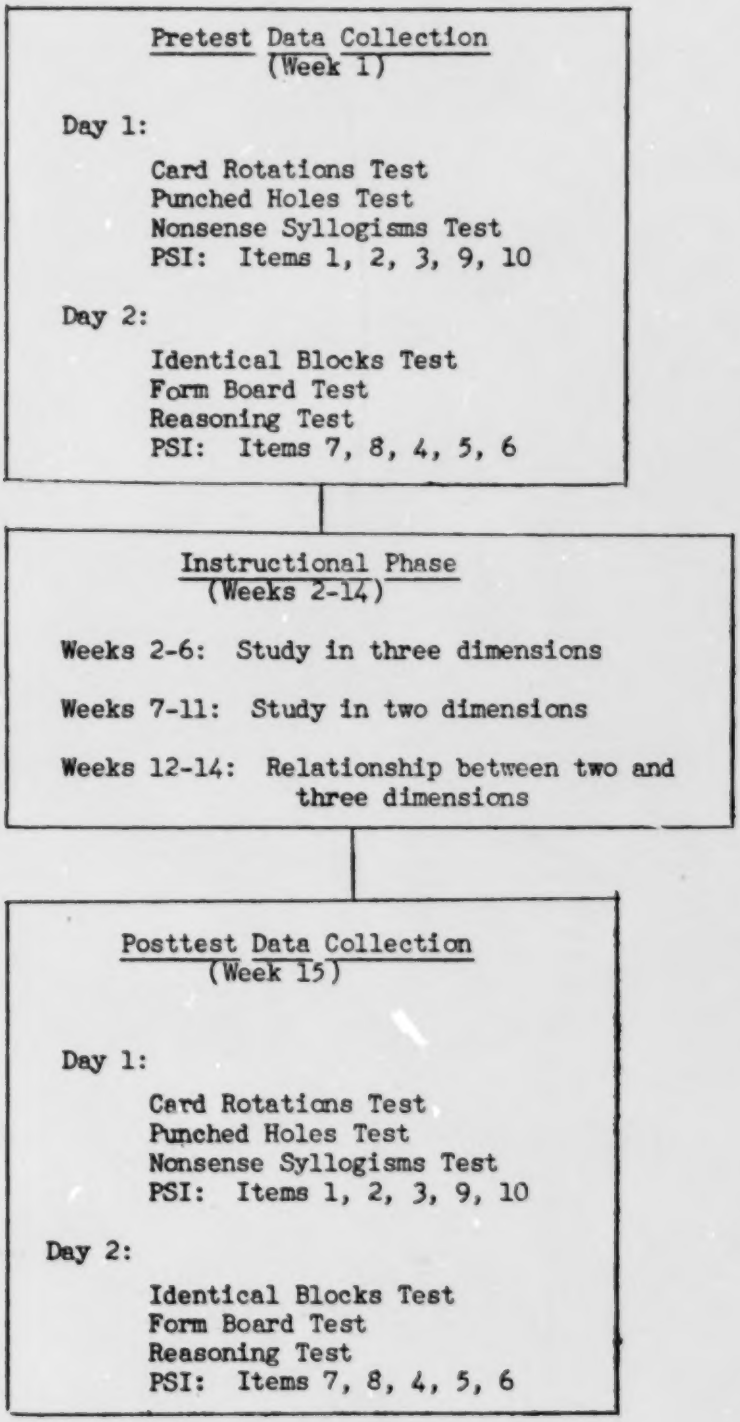

Figure 1: Outline of the Study

mathematics period. The control classes continued with their normal mathematics lessons during this time. During the final week of the investigation, the same evaluation instruments were administered to both the experimental and the control classes.

The pretest and posttest batteries consisted of the same set of seven tests. Four of the tests were spatial ability tests that had been selected based on a previous research study by the investigator. The four spatial tests chosen were Punched Holes Test, Card Rotations Test, Form Board Test and Identical Blocks Test. Two of the tests in the battery were logical reasoning tests which were selected based on pilot-testing. These two tests were Nonsense Syllogisms Test and Reasoning Test. The seventh test was a Problem-Solving Inventory compiled by the investigator which consisted of ten non-routine mathematics problems. A brief description of each test, accompanied by a sample item, follows.

1. Punched Holes Test

This test is part of the Kit of Reference Tests for Cognitive Factors. The modified form for fifth-grade students is from the NLSMA battery. The purpose of the test is to measure the ability to manipulate mentally given spatial patterns into other visual configurations. In each item, a square sheet of paper is shown folded two or three times, and a hole is then punched through all the thicknesses. Each of the folding moves as well as the punching move is illustrated in a separate picture. The individual must then choose which of five squares with holes in them would be the actual

representation of the original paper when unfolded. A sample item is shown in Figure 2.

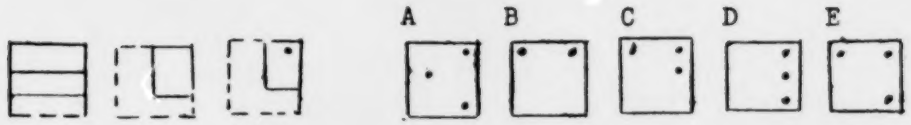

Figure 2. Item from Punched Holes Test

2. Card Rotations Test

This test is part of the Kit of Reference Tests for Cognitive Factors. The modified form for fifth-grade students is from the NLSMA battery. The purpose of the test is to measure the ability to perceive mentally the relationships among the elements of a figure in order to identify the figure when its orientation has been changed. In each item, there is a given irregular configuration. Then eight other illustrations of the same configuration are shown, some of which are rotated and some of which are flipped over. The individual must select those configurations which are rotations of the original configuration. A sample item is given in Figure 3.

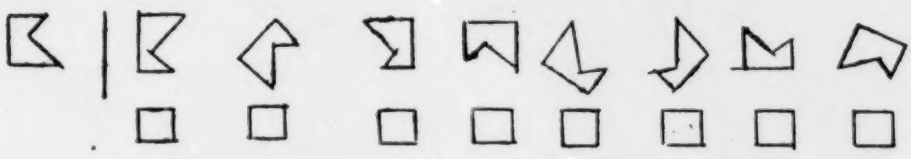

Figure 3. Item from Card Rotations Test

3. Form Board Test

This test is part of the Kit of Reference Tests for Cognitive Factors. The modified form for fifth-grade is part of the NLSMA battery. The purpose of the test is to measure the ability to manipulate mentally one spatial pattern into other visual configurations. In each item, there is a picture of a square with a piece missing. The task is to determine which two of five similar pieces would be used to complete the square. A sample item is shown in Figure 4.

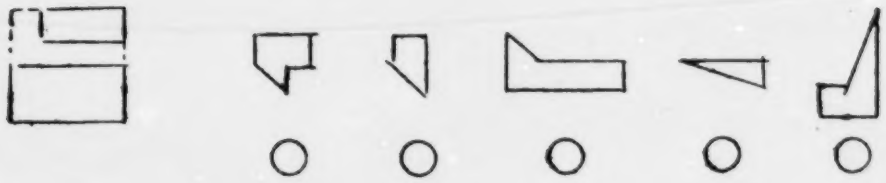

Figure 4. Item from Form Board Test

4. Identical Blocks Test

This test was constructed by Gulliksen. The purpose of the test is to measure the ability to perceive mentally the relationships among the elements of a three-dimensional figure in order to identify the figure when its orientation has been changed. In each item, a three-dimensional configuration is shown, followed by five similar configurations. The individual must select the one configuration which is a rotation of the original illustration. A sample item is shown in Figure 5.

Figure 5. Item from Idential Blocks Test

5. Nonsense Syllogisms Test

This test is part of the Kit of Reference Tests for Cognitive Factors. The purpose of the test is to measure the ability to reason from given to a conclusion. In each item, there are two premises having nonsensical content. Then a conclusion is stated. The individual must decide if the conclusion follows from the premises. A sample item is shown in Figure 6.

All elephants can fly. All giants are are elephants. Therefore all giants can fly.

Figure 6. Item from Nonsense Syllogisms Test

6. Reasoning Test

This test was constructed by Werdelin. The purpose of the test is to measure the ability to reason logically from given premises. (The items are not of nonsensical content.) In each item, there

are two given premises and a conclusion. The individual must decide if the conclusion is true, false or indeterminate. A sample item is shown in Figure 7.

> Carl is older than Bert and Peter is younger than Bert. Is Carl older than Peter?

Figure 7. Item from Reasoning Test

7. Problem-Solving Inventory.

This test was constructed by the investigator (see Appendix A). The purpose of the test is to measure problem-solving performance and solution processes. The test consists of ten non-routine mathematics problems. The individual was instructed to show all work in solving the problems.

Four scores were computed for each student from both the pretest and posttest batteries:

(a) Spatial ability score:

Spatial ability was defined as the ability to perceive the essential relationships among the elements of a given visual situation and the ability to manipulate mentally one or more of these elements. The spatial ability score was omputed by summing the z-scores on the four spatial tests.

(b) Reasoning ability score:

Reasoning ability was defined as the ability to analyze, step-by-step, certain premises and deduce conclusions. The reasoning ability score was computed by summing the z-scores on the two logical reasoning tests.

(c) Problem-solving performance score:

Problem-solving performance was defined as the ability to determine the outcome of a problem. (A problem is a situation where a procedure for determining the outcome is not immediately known.) The problem-solving performance score was computed by counting the number of correct responses to the ten problems on the Problem-Solving Inventory.

(d) Degree of visuality score:

Degree of visuality is the extent to which the subject uses visual solution processes (i.e., processes involving visual images such as diagrams, drawings, tables, and graphs) to solve a given mathematics problem. The degree of visuality score was computed by assigning a score of 0, 1, or 2 to each item on the Problem-Solving Inventory (based on the amount of visual solution processes used) and then summing across the ten items.

The investigator was responsible for all of the instruction; this disallowed for any differences in results due to varied teach-

ing effectiveness between teachers. The instruction began with a five-week series of lessons with three-dimensional objects. Geometric models of cubes, pyramids, cylinders, cones, prisms and spheres were examined. Construction (and analysis) of three-dimensional figures was done with D-stix, pipe cleaners, clay and construction paper. Cross-sections were enumerated. Relevant and irrelevant characteristics of each model were discussed.

The next five-week series of lessons dealt with two-dimensional objects. During this period, activities were developed using tangrams, geoboards and construction paper. Relevant and irrelevant characteristics of common two-dimensional figures were discussed. The notions of congruence and similarity were demonstrated. Tessilation activities were performed.

The final three-week series of lessons emphasized the relationship between three-dimensional and two-dimensional figures. Discussion of the two-dimensional figures which make up different three-dimensional objects was a major part of this instructional phase. Drawings of three-dimensional objects were completed.

During the entire instructional period, no new terminology was introduced. All terms employed were suggested by the students themselves. Sensing, imaging and drawing (all of which are part of the spatial mode of thinking) were stressed, with verbal analysis kept to a minimum.

Results

The means and standard deviations of the raw pretest and post-test scores on the seven tests are summarized in Table 1.

Table 1. Descriptive Statistics

Test	Maximum Possible Score	Mean	S.D.
Punched Holes Test	10	4.92 (5.82)	1.87 (2.34)
Card Rotations Test	112	63.84 (85.43)	18.24 (20.11)
Form Board Test	24	7.21 (8.41)	3.02 (3.87)
Identical Blocks Test	24	4.82 (5.72)	2.89 (3.03)
Nonsense Syllogisms Test	15	4.14 (4.89)	2.34 (2.78)
Reasoning Test	20	7.09 (8.25)	4.68 (5.03)
Problem-Solving Inventory	10	2.15 (4.03)	1.27 (1.48)

Note: Posttest data are given in parentheses.

All of the test scores increased from the pretest administration to the posttest administration. In order to determine the effects of the spatial instruction on these increases, the control classes were compared to the experimental classes which had received the instruction. Analyses of covariance were performed using pretest measures as covariates, in order to factor out any initial differences between the groups. The results of these analyses are given in Tables 2-4.

Table 2

Instructional Effects on Spatial Ability

Source of Variation	SS	df	F
Spatial ability pretest (covariate)	462.21	1	94.31**
Group membership (control vs. experimental)	106.72	1	25.36*

** $p < .05$
* $p < .10$

Table 3

Instructional Effects on Reasoning Ability

Source of Variation	SS	df	F
Reasoning Ability Pretest (covariate)	422.67	1	92.31**
Group Membership (control vs. experimental)	97.32	1	19.57*

** $p < .05$
* $p < .10$

Table 4

Instructional Effects on Problem-Solving Performance

Source of Variation	SS	df	F
Problem-Solving Performance Pretest (covariate)	65.34	1	69.28*
Group Membership (control vs. experimental)	87.21	1	93.24**

** p < .05
* p < .10

Since it was discovered that the instruction did affect problem-solving performance, further analysis was necessary to explore this effect. In particular, the effect of the instruction on spatial problems alone (#1, 4, 6, 7) and on analytic problems (#2, 5, 9) alone was explored. Two separate analyses of covariance were performed: the first measured instructional effects on the spatial problems and the second measured instructional effects on the analytic problems. These results are found in Tables 5 and 6.

Table 5

Instructional Effects on Spatial Problems

Source of Variation	SS	df	F
Spatial Problems Pretest (covariate)	21.34	1	51.27*
Group Membership (control vs. experimental)	35.11	1	86.92**

** p < .05
* p < .10

Table 6

Instructional Effects on Analytic Problems

Source of Variation	SS	df	F
Analytic Problems Pretest (covariate)	5.27	1	20.89*
Group Membership (control vs. experimental)	1.08	1	4.06

* $p < .10$

An aptitude-treatment interaction was also considered. What role did spatial ability play in this instructional effect? Was the instruction helping low spatial ability students more than high spatial ability students in terms of improvement in problem-solving performance? Was the instruction helping low reasoning ability students more than high reasoning ability students with respect to this same variable? Two separate t-tests were performed on the data from only those students who received instruction (the experimental classes only) in order to determine if the amount of spatial ability and/or the amount of reasoning ability of a student was significant in predicting problem-solving performance. The results of these analyses are presented in Tables 7 and 8.

Table 7

Instructional Effects on Problem-Solving Performance
Gain Score for High Spatial
Ability Versus Low Spatial Ability Students

	Mean	S.D.	t-value
High Spatial Ability Students	1.72	.93	.68
Low Spatial Ability Students	1.89	.98	

Table 8

Instructional Effects on Problem-Solving Performance
Gain Score for High Reasoning
Ability Versus Low Reasoning Ability Students

	Mean	S.D.	t-value
High Reasoning Ability Students	1.42	.73	2.5**
Low Reasoning Ability Students	1.97	.94	

** $p < .01$

Another issue to be considered was the importance of the sex-treatment interaction. Did the females profit more from the instruction than the males? A t-test was performed with data from students in the experimental classes only. The results of this analysis are presented in Table 9.

Table 9

Instructional Effects on Problem-Solving Performance
Gain Score for Males Versus Females

	Mean	S.D.	t-value
Males	1.83	.94	.24
Females	1.89	.96	

A final analysis concerned itself with the instructional effects on solution processes. Did the instruction affect the use of visual solution processes (involving diagrams, tables, graphs, and pictures)? An analysis of covariance was performed with degree of visuality as the covariate. The results of this analysis are shown in Table 10.

Table 10

Instructional Effects on Degree of Visuality

Source of Variation	SS	df	F
Degree of Visuality Pretest (covariate)	408.71	1	61.42*
Group Membership (control vs. experimental)	8.25	1	1.03

* $p < .10$

Conclusion and Discussion

The importance of spatial instruction in the problem-solving process must be recognized after noting many of the statistical results in the previous section. The role of spatial instruction is manifold.

1. The spatial instruction significantly affected mathematical problem-solving performance; the effect was more significant on problems which were more spatial in nature than on problems which were more analytic in nature.

These findings appear to confirm the notion that spatial instruction is a lacking element in the teaching of mathematical problem solving and can be taught effectively in the classroom.

2. The instruction did not significantly affect the solution processes used in problem solving, as manifested by written work. Further investigation by individual interviews, however, showed that students were using spatial techniques (e.g. drawings, graphs, tables) to a greater extent after the instruction, but they did not write these processes on their papers. (This may be due to prior training.)

3. There were no significant differences in instructional effects in problem-solving performance between students of high and low spatial ability. However, there were significant differences in instructional effects in problem-solving performance between students of high and low logical reasoning ability; students of low reasoning ability profited more from the instruction.

These findings seem to strengthen the conclusion stated in finding #1. The student who has found no success in an analytic mode of instruction has no other tools upon which to rely. Supplying such a student with spatial skills greatly increases his probability of success on problem-solving activities.

4. There were no significant differences in instructional effects in problem-solving performance between males and females.

This finding contradicts previous research which indicated that females are lower in spatial ability and problem-solving performance due to their lack of experience. Certain researchers (Fennema, 1977) had hypothesized that, given the proper training, females could be brought up to the level of males. This investigation did not confirm this hypothesis.

Summary and Implications

It appears that solving complex mathematics problems often requires mentally manipulating the data; therefore the ability to visualize the problem is essential.

The positive effects produced by the series of spatial lessons of this investigation reinforce the belief that such exercises are lacking in the regular mathematics curriculum. Spatial ability is not innate; rather, the proper instruction in a spatial mode of thinking can remove deficiencies and increase this aptitude.

As a mode of thinking, spatial thinking has been sorely ignored by most subject areas. Future studies should concern themselves with the effects of lessons in a spatial mode in other areas, such

as the natural sciences, social studies, English, etc. Also, long-term results should be analyzed. If a series of spatial lessons is presented year after year, will students have a greater tendency to rely on spatial techniques in solving problems? Will they be less hesitant to demonstrate these processes on their test papers?

Can the regular mathematics curriculum be altered in such a way that the normal material is presented in a spatial manner rather than including the spatial lessons as an additional topic? For example, could division be taught in a spatial mode?

Since the evidence has demonstrated that spatial instruction can positively affect problem-solving performance, the elementary school classroom teacher should begin to consider the use of such techniques in teaching mathematics.

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APPENDIX A
PROBLEM SOLVING INVENTORY

1. Suppose that there are four people in a race--John, Dick Steve and Tom. Suppose that John wins the race and Steve loses the race. If Tom is ahead of Dick, who came in second?

2. A farmer has pigs and chickens in his farm. There are 20 animals in all. If you count 56 feet in all, how many pigs and how many chickens does the farmer have?

3. Imagine a jar with a lid that is 6 cm. across. The jar is 8 cm. high. At the bottom of the jar, there is a caterpillar. Each day the caterpillar crawls up 4 cm. Each night he falls down 2 cm. How long will it take him to touch the lid of the jar?

4. Susan has 6 pockets and 15 dimes. She wants to put her dimes into her pockets so that each pocket contains a different number of dimes. How can she do this?

5. Bob had a pocketful of coins that added up to $1.60. The coins were nickels, dimes and quarters. If there were 9 coins in all, how many nickels, how many dimes, and how many quarters were there?

6. A fireman stood on the middle step of a ladder, directing water into a burning building. As the smoke got less, he climbed up three steps and continued his work. The fire got worse so he had to go down 5 steps. Later, he climbed up the last six steps and was at the top of the ladder. How many steps were there?

7. Imagine a doughnut sliced at (a).

(a)

Remove the section marked 1. Now pretend that you are looking at it from the <u>side</u> after it has been cut. Draw what you see.

Now take a doughnut and cut it at (b).

(b)

Remove the section marked 2. Now pretend that you are looking at it from the <u>side</u> after it has been cut. Draw what you see.

8. Mr. Jones traded his horse for 2 cows. Next he traded the 2 cows. For each cow, he got 3 pigs. Then he traded the pigs. For each pig, he got 3 sheep. Altogether, how many sheep did Mr. Jones get in the trade?

9. Suppose you are now 11 years old, and your mother is now 30 years old. In how many years will your mother be exactly twice as old as you?

10. There were 4 people in a room. Everyone shook hands with everyone else once. How many handshakes were there? If there were 5 people in the room, how many handshakes would there be?